**Problem Set 6 Answers**

**Problem 1**. Recall Problem 5 from Homework 5. That is:

A piston in a heart valve must fit into a sleeve. Due to inevitable variations in manufacturing processes, the diameters of pistons and sleeves vary somewhat. The fraction of the piston/sleeve assemblies meeting the required tolerance is only 52%.

To improve the yield, management has decided to adopt a strategy of sorting pistons and sleeves in batches of size five. That is, they will wait until they have produced 5 pistons and five sleeves. They will then measure and sort the pistons from largest to smallest and the same thing with the sleeves. The largest piston will be matched with the largest sleeve, the next largest piston with the next largest sleeve, and so on. Whatever else you might think of this manufacturing tactic (it is actually used in industry), one thing of interest is whether or not it improves the process yield. To answer this question, management took a 20 samples of 5 pistons and 5 sleeves each (for a total of 100 piston-sleeve pairs). Each group of five was sorted and matched up as described immediately above. Afterwards, the resulting clearances were measured, and it was found that 79 out of the 100 assemblies met the required tolerance.

1. Management wishes to perform an hypothesis test to determine whether or not the process yield has actually been improved from the 52% level. What is the appropriate null hypothesis? Hint: taking action should be associated with rejecting the null hypothesis.

**Null Hypothesis: yield of new process < 52%**

1. Compute the test statistic. Hint: this is a problem dealing with proportions.

**Our sample proportion, P = 79% & the hypothesized proportion is 52%. We can compute the standard error under the null hypothesis and the test statistic:**



1. Suppose we want a confidence level of 99.99%. Using a Z-test, determine the rejection region.

**normsinv(.9999) = 3.71947**

**Reject the null hypothesis if: Test statistic > 3.71947**

1. Based on your answers to parts b and c, should you reject the null hypothesis?

**YES!**

1. What does this mean operationally for management of the heart-valve company?

**They should move to institute selective assembly.**

1. Compute the p-value corresponding to the test statistic.

**We got 79%---what is the probability of this occurring if the underlying distribution of X is normal, with mean = .52 & σ = .04996?**

**In other words, we seek P{X >= 0.79} = 1 – P{X <= .79}**

**In Excel Language:**

**= 1 – normdist(.79,.52,.04996,1) = 3.26 x 10-8**

Problems 2-7 are somewhat repetitious. I have included them all to give you a “complete catalog of all possible options”. You should try working 1 or two of these on your own and then make certain you go over the answer sheet to understand how they are all done.

**Problem 2.** Suppose your null hypothesis in a Z-test is that the true average is < 100. You take a sample of size 100 and compute the test statistic. What is the associated p-value and should you reject the hypothesis at the 95% confidence level if:

**For problems 2 -7: Reject if our p-value is <= 0.05, otherwise Fail to Reject.**

Test statistic = 1.5

**1 – normsdist(1.5) = .067 (approx.)**

Test statistic = -1.5

**1 – normsdist(-1.5) = .933 (approx.)**

Test statistic = 2.5

**1 – normsdist(2.5) = .006 (approx.)**

Test statistic = -2.5

**1 – normsdist(-2.5) = .994 (approx.)**

**Problem 3.** Repeat problem 2, except that now you are doing a T-test instead of a Z-test.

Test statistic = 1.5

**tdistdist(1.5, 99, 1) = .068 (approx.)**

Test statistic = -1.5

**1 - tdistdist(1.5, 99, 1) = .932 (approx.)**

Test statistic = 2.5

**tdistdist(2.5, 99, 1) = .007 (approx.)**

Test statistic = -2.5

**1 - tdistdist(2.5, 99, 1) = .993 (approx.)**

**Problem 4.** Repeat problem 2, except that now the null hypothesis is that the true average > 100.

Test statistic = 1.5

**normsdist(1.5) = .933 (approx.)**

Test statistic = -1.5

**normsdist(-1.5) = .067 (approx.)**

Test statistic = 2.5

**normsdist(2.5) = .994 (approx.)**

Test statistic = -2.5

**normsdist(-2.5) = .006 (approx.)**

**Problem 5.** Repeat problem 4, except that now you are doing a T-test instead of a Z-test.

Test statistic = 1.5

**1 - tdistdist(1.5, 99, 1) = .932 (approx.)**

Test statistic = -1.5

**tdistdist(1.5, 99, 1) = .068 (approx.)**

Test statistic = 2.5

**1 - tdistdist(2.5, 99, 1) = .993 (approx.)**

Test statistic = -2.5

**tdistdist(2.5, 99, 1) = .007 (approx.)**

**Problem 6**. Repeat problem 2, except that now the null hypothesis is that the true average = 100.

Test statistic = 1.5

**Normsdist(-1.5) + [1 - normsdist(1.5)] = .134 (approx.)**

Test statistic = -1.5

**Normsdist(-1.5) + [1 - normsdist(1.5)] = .134 (approx.)**

Test statistic = 2.5

**Normsdist(-2.5) + [1 - normsdist(2.5)] = .012 (approx.)**

Test statistic = -2.5

**Normsdist(-2.5) + [1 - normsdist(2.5)] = .012 (approx.)**

**Problem 7.** Repeat problem 6, except that now you are doing a T-test instead of a Z-test.

Test statistic = 1.5

**Tdist(1.5, 99, 2) = .137 (approx.)**

Test statistic = -1.5

**Tdist(1.5, 99, 2) = .137 (approx.)**

Test statistic = 2.5

**Tdist(2.5, 99, 2) = .014 (approx.)**

Test statistic = -2.5

**Tdist(2.5, 99, 2) = .014 (approx.)**

**Problem 8. Hypothesis Testing**

1. In a t-test for the mean, your sample mean is 100, the null hypothesis is H0: true mean >= 50 and your sample standard deviation is 25. Compute the test statistic and the critical value if you have enough information to do so. Do you reject the null hypothesis at the 95% level, do you fail to reject the null hypothesis at the 95% level, or do you need additional information to tell?

**Not enough information. We need the sample standard error and, since it is not given to us in the problem, we would need to compute it. To do so, we need to know the sample size which we do not.**

1. In a t-test for the mean, your sample mean is 100, the null hypothesis is H0: true mean <= 50, your sample standard deviation is 50 and your sample size is 25. Compute the test statistic and the critical value if you have enough information to do so. Do you reject the null hypothesis at the 95% level, do you fail to reject the null hypothesis at the 95% level, or do you need additional information to tell?

**Sample standard error = 50/sqrt(25) = 10**

**Test Statistic = (100 – 50)/10 = 5**

**Reject if the test statistic is larger than tinv(.1, 24) = 1.71 (approx.)**

**Reject since test statistic of 5 is larger than the critical value of 1.71.**

1. In a t-test for the mean, your sample mean is 25 and your null hypothesis is H0: true mean = 25. Do you reject the null hypothesis at the 95% level, do you fail to reject the null hypothesis at the 95% level, or do you need additional information to tell?

**Even though we know neither the sample standard deviation nor the sample size, we can still compute the test statistic in this special case since the sample mean = hypothesized mean. Thus, the test statistic = 0. We should FAIL TO REJECT.**

1. In a t-test for the mean, your sample mean is 40, the null hypothesis is H0: true mean >= 50, the sample size is 30, and your sample standard deviation is 25. Compute the test statistic and the critical value if you have enough information to do so. Do you reject the null hypothesis at the 95% level, do you fail to reject the null hypothesis at the 95% level, or do you need additional information to tell?

**SSE = SSD/Sqrt(sample size) = 25/sqrt(30) = 4.56 (approx.)**

**Test Statistic = (40 – 50)/4.56 = -2.19 (approx.)**

**Rejection Region:**

**Reject if test statistic is < critical value = - tinv(.1, 29) = - 1.70 (approx.)**

**We should reject the null hypothesis.**

1. You have exactly the same information as in part d. Can you compute the p-value? If so, do it.

**p-value = tdist(2.19, 29, 1) = 0.018 (approx.)**

**Problem 9.** A particular disease that, up till now, has been untreatable, has an average survival length (time from initial diagnosis until death) of 9.25 years. A potential new treatment has been developed. Data from a sample of patients given the potential new treatment is given in the accompanying excel spreadsheet.

1. What is the average survival length of the patients in this sample?

**10.392**

1. What is the sample standard deviation?

**2.56 (approx.)**

1. What is the sample standard error?

**0.256 (approx.)**

**Problem 10.** Same data as from problem 9. Perform an hypothesis test to test whether or not the new treatment will lengthen the lifespan of patients receiving it. Be certain to specify your null hypothesis, compute the test statistic, compute the associated p-value, determine the rejection region, and draw your conclusion.

**Null Hypothesis: treated patients survive, on average, less than 9.25 years**

**I shall use a 95% significance level.**

**Test statistic = (10.392 – 9.25)/0.256 = 4.41**

**Rejection Region: Reject if test statistic > critical value = tinv(.1, 99) = 1.66 (approx.)**

**Conclusion: since 4.41 > 1.66, we should reject the null hypothesis----it looks like the treatment works.**

**p-value = tdist(4.41, 99, 1) = 1.3 x 10 -5 (approx.)**